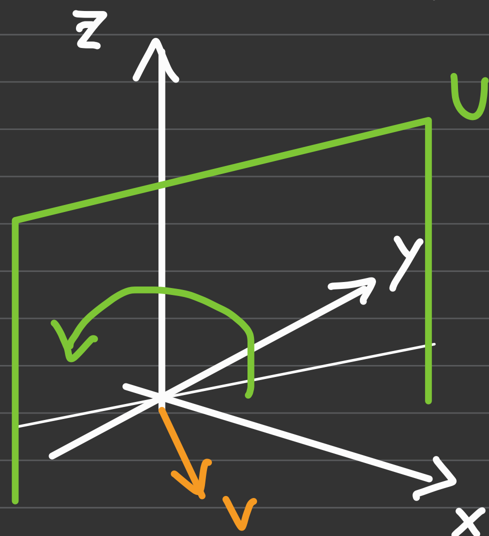
$\boxed{14}$ $v = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ $U = \mathrm{Span}(v)^{\perp} = \{x - y = 0\} =$

$$= \mathrm{Span}\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right)$$



$$\mathcal{B} = (v, \mathcal{B}_U)$$

$$\Longrightarrow [f]^{\mathcal{B}}_{\mathcal{B}} = \left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & & \\ 0 & \multicolumn{2}{c}{[f|_U]^{\mathcal{B}_U}_{\mathcal{B}_U}} \end{array}\right)$$

f rot attorno a v di angolo $\pi/2$

g rifl risp a U

$[f|_U]^{\mathcal{B}_U}_{\mathcal{B}_U}$ è ortogonale se $\mathcal{B}_U$ è ortonormale

$$B_U = \left\{ \underset{u_1}{\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}}, \underset{u_2}{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}} \right\}$$ è ortonormale

Affinché (v, u_1, u_2) sia destrogira verifichiamo che è $u_1 \times u_2$

$$u_1 \times u_2 = \det \begin{pmatrix} e_1 & e_2 & e_3 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{pmatrix}$$

ha lo stesso verso di $v = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \Longrightarrow B_U$ va bene

$$[f]_{B}^{B} = \left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{array}\right) = \left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & 0 & -1 \\ 0 & 1 & 0 \end{array}\right)$$

$$[g]_{B}^{B} = \left(\begin{array}{c|cc} -1 & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right)$$

$$[g \circ f]_{B}^{B} = [g]_{B}^{B} [f]_{B}^{B} = \left(\begin{array}{c|cc} -1 & 0 & 0 \\ \hline 0 & 0 & -1 \\ 0 & 1 & 0 \end{array}\right)$$

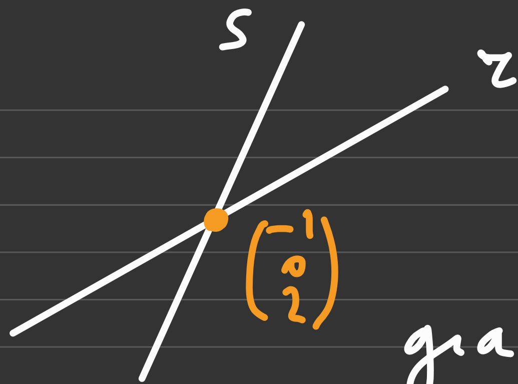
e si cambia base

$\boxed{1}$ $r, s \subseteq \mathbb{R}^3$ $\quad r \begin{cases} x = -1 \\ z = 2 \end{cases} \quad s \begin{cases} 2x + y - 2z = -6 \\ y + z = 2 \end{cases}$

Determinare la pos reciproca di r e s e il piano che li contiene (se $\exists$)

$$r \cap s \quad \begin{cases} x = -1 \\ z = 2 \\ 2x + y - 2z = -6 \\ y + z = 2 \end{cases} \qquad \begin{cases} x = -1 \\ z = 2 \\ -2 + 0 - 4 = -6 \\ y = 0 \end{cases}$$

$\Longrightarrow$ $r \cap s = \left\{ \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \right\}$ e quindi r e s sono incidenti $\Longrightarrow$ esiste il piano π che li contiene



$$\mathrm{giac}(r) = \{x = 0,\ z = 0\} = $$
$$= \mathrm{Span}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\mathrm{giac}(s) = \{2x + y - 2z = 0,\ y + z = 0\} = $$
$$= \mathrm{Span}\begin{pmatrix} -\frac{3}{2} \\ 1 \\ -1 \end{pmatrix} = \mathrm{Span}\begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$$

$$\Rightarrow \mathrm{giac}(\pi) = \mathrm{Span}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}\right)$$

$$\Rightarrow \pi = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} + \mathrm{Span}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}\right) \quad \text{eq param}$$

$$\mathrm{giac}(\pi) = \mathrm{Span}\left(\begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}3\\-2\\2\end{pmatrix}\right) = \{2x - 3z = 0\}$$

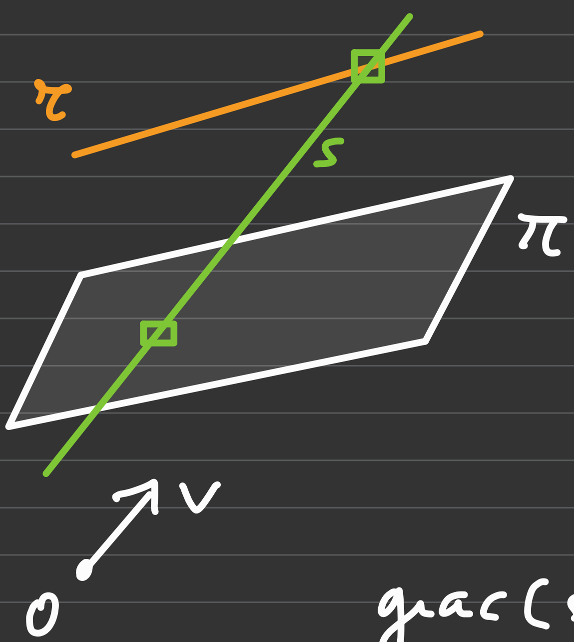
$$\Rightarrow \quad \pi = \{\underbrace{2x - 3z = -8}\}$$

il termine noto è -8
perché $\begin{pmatrix}-1\\0\\2\end{pmatrix} \in \pi$

ESERCIZIO provare a trovare π come piano per 3 pti

$\boxed{2}$

$\pi \quad x+y+z=1 \qquad r \quad \begin{pmatrix}2\\0\\1\end{pmatrix} + \mathrm{Span}\begin{pmatrix}1\\-1\\0\end{pmatrix}$



$v = \begin{pmatrix}1\\0\\0\end{pmatrix}$

Det le eq della retta s che è parallela a v e che interseca π e r

$$\mathrm{giac}(s) = \mathrm{Span}(v) = \mathrm{Span}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\Longrightarrow s = \begin{pmatrix}x_0\\y_0\\z_0\end{pmatrix} + \mathrm{Span}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$r = \left\{ \begin{pmatrix} 2+\lambda \\ -\lambda \\ 1 \end{pmatrix} \quad \lambda \in \mathbb{R} \right\}$$ e scegliamo come

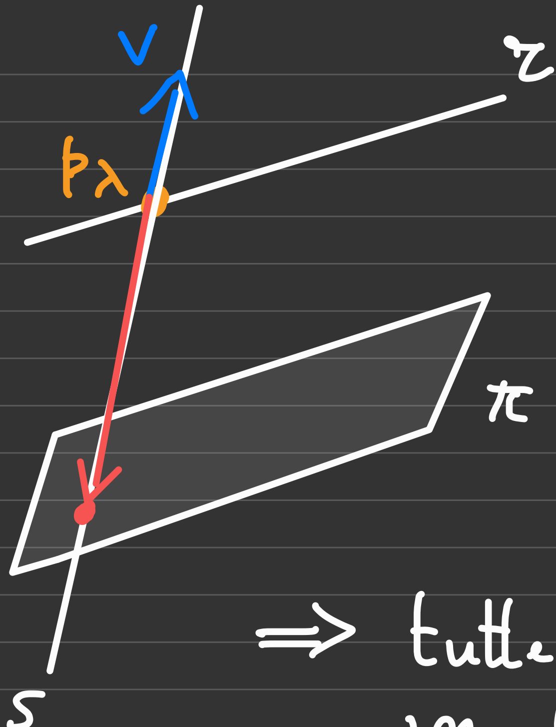
$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$ un pto di r $\longrightarrow$ P_λ (vedi disegno a pag seguente)

$$\Longrightarrow \quad s = \begin{pmatrix} 2+\lambda \\ -\lambda \\ 1 \end{pmatrix} + \mathrm{Span} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} =$$

$(\star)$

$$= \left\{ \begin{pmatrix} 2+\lambda+\mu \\ -\lambda \\ 1 \end{pmatrix} \quad \mu \in \mathbb{R} \right\}$$

$$s \cap \pi \neq \emptyset \iff \begin{pmatrix} 2+\lambda+\mu \\ -\lambda \\ 1 \end{pmatrix} \in \pi \text{ per qualche } \mu \in \mathbb{R}$$



$$\begin{pmatrix} 2+\lambda+\mu \\ -\lambda \\ 1 \end{pmatrix} \in \pi$$

$$\Updownarrow$$

$$2+\cancel{\lambda}+\mu-\cancel{\lambda}+1 = 1$$

$$\mu = -2 \quad \forall \lambda \quad (!)$$

$\Rightarrow$ tutte le rette s costruite in ($\star$) intersecano π perché $\forall \lambda$ si trova μ