EXERCISES OF WEEK ONE (2014/09/22, 11:00AM)

Exercise 1. For each of the following sentences, state whether they are true or find a counterexample

1. if $n = 1$, for every v, w in E_1 there holds

$$v \cdot w = 0 \Rightarrow v = 0 \text{ or } w = 0$$

2. the same statement as in 1. except that $n \geq 2$.

When $n = 2$, given $v = (v_1, v_2)$ in E_2, we define

$$v^\perp := (v_2, -v_1).$$

In the next questions the linear space is E_2.

3. $\|v\| = \|v^\perp\|$

4. given v, w such that $w \neq 0$, we have

$$|v \cdot w| + |v \cdot w^\perp| = 0 \Rightarrow v = 0$$

5. write the quantity

$$|v \cdot w|^2 + |v \cdot w^\perp|^2$$

in terms of $\|v\|$ and $\|w\|$

6.

$$|v \cdot w| = |v \cdot w^\perp| \Rightarrow v = w + w^\perp.$$

Exercise 2. Prove the Cauchy-Schwarz inequality

$$|v \cdot w| \leq \|v\| \|w\|$$

for every $v, w \in \mathbb{R}^n$ (start by squaring both terms $|v \cdot w|$ and $\|v\| \|w\|$. It can be easier to look at the case $n = 2$ before the general case).

Date: 2014, September 13.