EXERCISES OF WEEK FOUR (2014/10/07, 11:00AM)

Exercise 1. Suppose that we have two non-degenerate planes in $\mathbb{R}^3$

$$\pi := \pi(P, u, v), \quad \pi' := \pi(Q, w, z).$$

We can define a distance between π and π' as

$$d(\pi, \pi') := \inf\{d(R, R') \mid R \in \ell, R' \in \ell'\}.$$

Try to express the distance in terms of P, Q, u, v, w, z.

Exercise 2. Suppose that you have a line in $\ell \subseteq \mathbb{R}^2$ given by the equation

$$\ell : ax + by = c.$$

Find a formula for the distance of a point $P(x_0, y_0)$ from ℓ in terms of a, b, c.

Exercise 3. Find the intersection of the two planes given in Cartesian form

$$\pi_1 : 2y + 3y + z = 1 \tag{1}$$

$$\pi_2 : 2y + 3y + 2z = -2. \tag{2}$$

Exercise 4. Given two lines in $\mathbb{R}^2$, $\ell(P, v) \neq \ell(Q, w)$ we know that the intersection is non-empty if and only if $v \times w \neq 0$.

Now, suppose that we have three non-degenerate lines in the plane

$$\ell_1 := \ell(P, v), \quad \ell_2 := \ell(Q, w), \quad \ell_3 := \ell(R, z)$$

such that

$$\ell_j \neq \ell_k \text{ for every } j \neq k$$

find conditions on P, Q, R, v, w, z such that

$$\ell_1 \cap \ell_2 \cap \ell_3 \neq \emptyset.$$

Date: 2014, October 1.