EXERCISES OF WEEK FOUR

README. The due date for this assignment is Tuesday, October 7th. If you cannot come to class, please, send me the assignment by e-mail by 1pm. Try to do at least four exercises. Please contact me, for *any* question.

Exercise 1. For each of the following differential equation, write a normal form and its domain.

Also, check whether the function is a solution (Sol.) to the corresponding differential equation (Eq.)

$$Sol.: (e^{2x}, (0,1)) \qquad Eq.: 4y''(x) - y(x) = 0 \tag{1}$$

$$Sol.: (\sqrt{1-x}, [0,1]) \qquad Eq.: 2y(x)y'(x) = -1 \tag{2}$$

$$Sol.: (e^{x^2/2}, (-\infty, +\infty)) \qquad Eq.: y'(x)/x = y(x) \tag{3}$$

$$Sol.: (x^2, (-\infty, +\infty)) \qquad Eq.: y'(x) = 2\sqrt{y(x)}. \tag{4}$$

Exercise 2. Integrate each of the following differential equations

$$y'(x) = y(x)(1 - y(x)) \tag{5}$$

$$y'(x) + 2xy^2(x) = 0. \tag{6}$$

Among the solutions of (5) find at least three solutions with existence interval $\mathbb{R}$. Among the solutions of (6) find at least one solution such that the existence interval is not $\mathbb{R}$.

Exercise 3. Let g and f be two derivable Lipschitz functions on the interval $[0,1]$. Is fg a Lipschitz function?

Exercise 4. Let y be a one-variable function which is 1 on the interval $(0,1)$ and 2 on the interval $(1,2)$. Is it Lipschitz?

Exercise 5. Check whether each of the following functions are Lipschitz or locally Lipschitz (if it is locally Lipschitz, write explicitly what is r in $Q_r(x_0, y_0)$)

$$g_1\colon (0,1) \times (0,1) \to \mathbb{R}, \quad g_1(x,y) = \sin(1/x) \tag{7}$$

$$g_2\colon \mathbb{R} \times [0, 4\pi] \to \mathbb{R}, \quad g_2(x,y) = |\sin y| \tag{8}$$

$$g_3\colon \mathbb{R} \times \mathbb{R} \to \mathbb{R}, \quad g_3(x,y) = xy(1-y) \tag{9}$$

$$g_4\colon (1,2) \to \mathbb{R}, \quad g_4(x) = \frac{|x-1|}{x}. \tag{10}$$

Exercise 6. Let $(y, (0,1))$ be a solution to the differential equation

$$y'(x) = y(x)\sin y(x)$$

such that $y(0) = \pi/2$. Show that $0 < y(x) < \pi$ for every $0 \le x \le 1$.

Date: 2014, September 26.