

EXERCISES OF WEEK SIX

Exercise 1 (Check problems, 1,2, 3 and 4 at page 59 of "Advanced Engineering Mathematics"). For each of the following equations, check whether they are exact or not. If they are exact, find an explicit solution

(1) $(2x - 1)dx + (3y - 7)dy = 0$

(2) $(2x + y)dx - (x + 6y)dy = 0$

(3) $(5x + 4y)dx + (4x - 8y^3)dy = 0$

(4) $(\sin y - y \sin x)dx + (\cos x + x \cos y - y)dy = 0.$

Moreover,

1. in (1), find all the solutions y defined on $(-\infty, +\infty)$ such that

$$y(x) > -\frac{7}{3}$$

for every x in $(-\infty, +\infty)$

2. in (3): is there a solution (y, I) such that $0 \in I$ and $y(0) = -1/2$ (you do not have to find this solution explicitly, just give a reason why this solution exists or not)

3. in (4), is there a solution such that $y(\pi) = \pi$?

Exercise 2. Check whether the following differential equation

$$(1 - y) \cos x + (2y - 1 - \sin x)y' = 0$$

is exact. Moreover, for every $0 \leq k \leq 3$ find a solution y_k such that $y_k(0) = 0$ and

$$\int_{\pi/2}^{13\pi/2} y_k(x) dx = 2k\pi.$$