SOLUTIONS OF THE ASSIGNMENT OF WEEK FOURTEEN

Exercise (Exercise 4, Week Thirteen)**.**

(i) If $a = b$, then $S_a = S_b$.
(ii) is it true that if $S_a = S_b$, then $a = b$?

Solution.

(i) $x \in S_a \Rightarrow x < a \Rightarrow (x, a) \in R$ and $x \neq a$. Since $a = b$, $(x, a) = (x, b)$ and $x \neq b$. Hence, $x < b$
(ii) in general, it is not true. For instance, let

$$A = \{0, 1, 2\}, \quad R := id_A \cup \{(0, 1), (0, 2)\}.$$

Then $S_1 = S_2 = \{0\}$, but $1 \neq 2$.

$\square$

Exercise (Exercise 3, page 98 of the book)**.** Let G be an equivalence relation on A. Then $G \circ G = G$.

Solution. Since G is an equivalence relation, it is transitive and $G \circ G \subseteq G$. We show the converse inclusion: let $(x, y) \in G$. Since G is reflexive, $(x, x) \in G$. Then, we can obtain (x, y) as composition of the two ordered pairs $(x, x), (x, y) \in G$. Then $(x, y) \in G \circ G$. $\square$

Exercise (Exercise 1, page 146 of the book)**.** Let $(A, \leq)$ be a f.o.c. (fully ordered class) define B the class

$$S \in B \Leftrightarrow S \text{ is a set and a section.}$$

Then, $(B, \leq)$ is a f.o.c with the order relation $S_1 \leq S_2 \Leftrightarrow S_1 \subseteq S_2$.

Solution. We argue by contradiction. Let S_1 and S_2 be two sections which are not comparable. Then, there are

$$y \in S_1 \cap S_2', \quad z \in S_2 \cap S_1'.$$

Since $(A, \leq)$ is a f.o.c, either $y \leq z$ or $z \leq y$. If $y \leq z$, then $z \in S_2 \Rightarrow y \in S_2$, because S_2 is a section, and we obtain a contradiction with $y \notin S_2$. Similarly, if $z \leq y$, we obtain a contradiction with $z \notin S_1$. $\square$

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