WEEK FIFTEEN - SOLUTIONS OF THE PROBLEMS OF THE EXAM

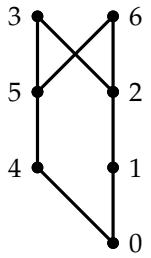
Exercise 1. Let $(A, \leq)$ be a p.o.c. Suppose that there are two maximal chains $C \neq D$. Then A is not a f.o.c.

Solution. We argue by contradiction. If A is a f.o.c, then A is a chain.
$C \subseteq A$. Since C is maximal, $C = A$.
$D \subseteq A$. Since D is maximal, $D = A$.
Then $C = D$, which contradicts $C \neq D$. □

Exercise 2. Is it true that $\mathscr{P}(2) = 4$? explain (and remember that $2 = \{0, 1\}$ and $4 = \{0, 1, 2, 3\}$)!

Solution. No, it is not true.
$\mathscr{P}(2) = \{0, \{0\}, 2, \{1\}\} = \{0, 1, 2, \{1\}\} \neq \{0, 1, 2, 3\} = 4$ because $3 \neq \{1\}$. □

Exercise 3. The following graph represents an order relation



Please, write

(1) the maximal chains
(2) the sections which contain 1 and 5
(3) how many different initial segments are there.

Solution.

(1) $\{0, 1, 2, 6\}$, $\{0, 1, 2, 3\}$, $\{0, 4, 5, 6\}$, $\{0, 4, 5, 3\}$
(2) $\{0, 1, 4, 5\}$, $\{0, 1, 4, 5, 2\}$, A, $A - \{6\}$, $A - \{3\}$
(3) we have

$$S_0 = 0, \quad S_1 = S_4 = 1, \quad S_5 = \{0, 4\}, \quad S_2 = 2, \quad S_3 = S_6 = 6 - \{3\}.$$

Thus, there are five different initial segments.

□

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