SOLUTIONS OF THE EXERCISES OF WEEK ONE

Exercise 1. Defining the set of the first 100,000 natural numbers as

$$A := \{1, 2, 3, \dots, 99999, 100000\}$$

is ambiguous. How could we define the set above with the Axiom of Unrestricted Comprehension Schema (find $p(n)$)?

Solution. We can use the UCCS Axiom with the property

$$p(n) : 1 \leq n \leq 100000.$$

Then, there exists

$$A := \{n \mid p(n)\}.$$

□

Exercise 2. Draw a truth table for $\neg(\neg P)$.

Solution.

P	$\neg P$	$\neg(\neg P)$
T	F	T
F	T	F

Since the columns one and three coincide, P is equivalent to $\neg(\neg P)$. □

Exercise 3. Can you show that the following sets are infinite?

(a) $\mathbb{Z} - \mathbb{N}$: relative integers which are not natural numbers
(b) $\mathbb{Q} - \mathbb{Z}$: rational numbers which are not relative integers
(c) $\mathbb{R} - \mathbb{Q}$: real numbers which are not rational numbers

Solution.

(a) for every $n \in \mathbb{N}$, we have
$$-n \in \mathbb{Z} - \mathbb{N}$$
then the set above is infinite
(b) for every $n \in \mathbb{N}$, we have
$$\frac{1}{n+1} \in \mathbb{Q} - \mathbb{Z}$$
then it is an infinite set
(c) for every $n \in \mathbb{N}$
$$n\sqrt{2} \in \mathbb{R} - \mathbb{Q}$$
then it is an infinite set.

□

Exercise 4. We defined $E := \{S \mid \#S = \infty\}$. We know that $E \in E$. Starting from E, can you find another set E_* such that $E_* \in E_*$?

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Solution. An example is given by

$$E_* := E - \{E\}.$$

We show that $E_* \in E_*$. Since $\#\{E\} = 1$ and $\#E = \infty$, we have $\#E_* = \infty$. Then

$$E_* \in E. \tag{1}$$

We already know that $E \in E$. Then $E \neq E - \{E\} = E_*$. Then

$$E \neq E_*. \tag{2}$$

By (1) and (2), it follows that $E_* \in E - \{E\}$. Then $E_* \in E_*$.

Another example is given by

$$D := \{S \mid \#S \geq 1\}.$$

Since $\{3\} \in D$, the set D has at least one element. Then $\#D \geq 1$. Therefore, $D \in D$. □