

SOLUTION OF THE EXERCISE OF WEEK THREE

Exercise. In the model

	x	y	z	A	B	C	D	E
x	1	0	0	1	0	1	1	0
y	0	0	1	0	1	1	1	0
z	0	1	0	1	1	0	1	0
A	0	0	0	0	0	0	0	0
B	0	0	0	0	0	0	0	0
C	0	0	0	0	0	0	0	0
D	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0

List all the

- (a) sets
- (b) proper classes
- (c) singletons
- (d) pairs which are not singletons
- (e) the universal class \mathcal{U}
- (f) the empty class \emptyset

for each of the given property $P(x)$, what is $\{x \mid P(x)\}$?

- (1) $P(x) : x \subseteq A$
- (2) $P(x) : x \subseteq B$
- (3) $C \cap A$.

Solution.

- (a) x, y, z
- (b) A, B, C, D, E
- (c) x, y, z
- (d) A, B, C
- (e) D
- (f) E

- (1) $A = \{x, z\}$. Then, the subclasses of A are

$$\emptyset, \{x\}, \{z\}, \{x, z\}.$$

All of them exist in the model and correspond to

$$E, x, y, A.$$

E and A are proper classes, then these are not included in $\{x \mid x \subseteq A\}$. Then

$$\{x \mid x \subseteq A\} = \{y, z\} = C.$$

The answer is C

(2) $B = \{y, z\}$. Then, the subclasses of A are

$$\emptyset, \{y\}, \{z\}, \{y, z\}.$$

All of them exist in the model and correspond to

$$E, z, y, B.$$

E and B are proper classes, then these are not included in $\{x \mid x \subseteq B\}$. Then

$$\{x \mid x \subseteq B\} = \{y, z\} = B.$$

The answer is B

(3) $C = \{x, y\}$ and $A = \{x, z\}$. Then $A \cap C = \{x\}$. The singleton of x exists in the model. In fact, $\{x\} = x$. Then $A \cap C = x$.

□