EXERCISES OF WEEK FIVE

Exercise 1. Suppose that the Subset Axiom holds. Then

$$\cap A$$

is a set for every class A such that $A \neq \emptyset$.

Solution. Since $A \neq \emptyset$, there exists x such that $x \in A$. Hence

$$\cap A \subseteq x.$$

By **A4**, $\cap A$ is a set. □

Exercise 2. Suppose that **A3** holds. Then

$$\cup \mathscr{U} = \mathscr{U}$$

where $\mathscr{U}$ is the Universal Class.

Solution. Since every class is a subclass of $\mathscr{U}$, we have

$$\cup \mathscr{U} \subseteq \mathscr{U}.$$

We show that

$$\mathscr{U} \subseteq \cup \mathscr{U}.$$

Let $x \in \mathscr{U}$. By ***A3***, the singleton $y := \{x\}$ is a set. Then $y \in \mathscr{U}$. Then

$$y \in \mathscr{U} \Rightarrow y \subseteq \cup \mathscr{U}.$$

Hence $x \in \cup \mathscr{U}$. □

Exercise 3. In model given below

$\in$	A	B	C	D
A	0	1	1	1
B	1	0	1	1
C	0	0	0	1
D	0	0	0	0

1. what are the order pairs?
2. does $\mathscr{U} \times \mathscr{U}$ exist?
3. what are the graphs?
4. is there a function $f\colon A \to A$?

Solution. 1. Ordered pairs: $A = (A, A)$ and $B = (B, B)$

2. the Universal Class $\mathscr{U} = \{A, B, C\}$ exists and is equal to D. The product $\mathscr{U} \times \mathscr{U}$ consists of all the ordered pairs. Then

$$\mathscr{U} \times \mathscr{U} = \{(A, A), (B, B)\} = \{B, A\} = C$$

3. by definition a graph is a subclass of $\mathscr{U} \times \mathscr{U}$. Then the graphs are

$$\emptyset, \{A\}, \{B\}, \{A, B\}$$

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that is

$$B, A, C$$

4. since A has only one element, the only possible function $f\colon A \to A$ is

$$f := \{(B, B)\} = \{B\} = A.$$

Then a function exists: A.

$\square$