

TEOREMA di SCHOTTY

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PICCOLO TEOREMA di PICARD

$f: \Delta_r \rightarrow \mathbb{C}$ olomorfa $\Delta_r = r\Delta$

def: Il numero di LANDAU per $f: \Delta_r \rightarrow \mathbb{C}$ olomorfa

$$L_f = \sup \{ r > 0 \mid f(\Delta_r) \supset \Delta_r \}$$

LEMMA: Dato $R > 0$, $f: \Delta_R \rightarrow \mathbb{C}$ olomorfa $f(0) = 0$

$$f(\Delta_R) \supseteq \frac{\Delta_R \cdot |f'(0)|^2}{4 \|f\|_\infty}$$

$$\mathcal{R} = \{ f: \Delta \rightarrow \mathbb{C} \text{ olom.} \mid f(0) = 0, f'(0) = 1 \}$$

def: Il numero di LANDAU

$$L = \inf \{ L_f : f \in \mathcal{R} \}$$

TEO: (BLOCH-LANDAU) $L \geq 4/16$, $f \in \Theta(\Delta)$, $f'(0) = 1$

LEMMA: $f: \Omega \rightarrow \Omega_{0,1}$ olomorfa $\Omega \subseteq \mathbb{C}$ sempl connesso
 $\Omega_{0,1} := \{z \mid \operatorname{Re} z \in (0,1)\} \Rightarrow \exists g \in \Theta(\Omega)$

$$f = -\exp(\pi i \cosh(2g))$$

1) $g(\Omega)$ non contiene nessun disco di raggio 1

2) $K \subset \subset \Omega_{0,1}$, $\exists C_0 := C_0(K) : f(\Omega) \subset K$

$$|g(z)| \leq C_0$$

oss: $f: \Omega \rightarrow \Omega_{0,1}$, $|\log|f|| = \pi \sinh(2|\operatorname{Re} g|) |\operatorname{Im}(z \operatorname{Im} g)|$

TEO: (SCHOTT) Dato $0 < \kappa < 1$, $a_0 \in \Omega_{0,1} \Rightarrow C_0 := C_0(\kappa, a_0)$

$\forall f: \Delta \rightarrow \Omega_{0,1}$ olomorfa, $\forall z \in \Delta_\kappa$, $f(z) = a_0$

$$|f(z)| \leq C_0$$

C_0 è limitata sui cpt di $(0,1) \times \Omega_{0,1}$

DM: $g \in \Theta(\Delta)$ $f = -\exp(\pi i \cosh(2g))$, $\kappa < R \leq 1$

$z_0 \in \Delta_\kappa : (g'(z_0)) \neq 0$

$$\psi(z) = \frac{g(z_0 + (1-\kappa/R)z)}{(1-\kappa/R)g'(z_0)}$$

1) ψ é olomorfa em Δ_R $z \in \Delta_R$

$$|z_0 + (1 - \pi/R)z| \leq |z_0| + \frac{1 - \pi}{R} |z|$$

$$\leq \pi + \frac{R - \pi}{R} R \leq R \leq 1$$

$$2) \psi'(z) = \frac{g'(z_0 + (1 - \pi/R)z) \cdot (1 - \pi/R)}{(1 - \pi/R) g'(z_0)}$$

$$\psi'(0) = 1$$

$$16 \leq \frac{1}{(1 - \pi/R) |g'(z_0)|}$$

$$|g'(z_0)| \leq \frac{16}{1 - \pi/R}$$

$\forall z \in \Delta_R$

$$|g(z) - g(0)| = \left| \int_0^z g'(\xi) d\xi \right| \leq \frac{16\pi}{1 - \pi/R}$$

$$|g(z)| \leq |g(0)| + \frac{16\pi}{1 - \pi/R} \quad R \rightarrow \infty$$

$$\forall z \in \Delta_\pi \quad |f(z)| \leq \exp\left(\pi \sinh(z \log 2) + \frac{32\pi}{1-\pi}\right) := C_{\log 2}$$

$$K \subset \subset [0, 1) \times \Omega_{0,1} \quad p_1, p_2$$

$$p_2(K) = K \subset \subset \Omega_{0,1}, C_0(K, \mathbb{R})$$

$$C_0(K, \mathbb{R}): p_2(K) \rightarrow \mathbb{R}_+$$

TEO: (LANDAU) Dati $(a_0, a_1) \in \mathbb{C} \times \mathbb{C}^* \Rightarrow \exists R(a_0, a_1) > 0 \exists \pi:$
 $\exists f \in \text{Hol}(\Delta_\pi, \Omega_{0,1})$

$$r \leq R(a_0, a_1)$$

TEO: (PICCOLO TEOREMA di PICARD) $f: \mathbb{C} \rightarrow \Omega_{a,b}$ omnia
 $\Rightarrow f$ è costante $\neq a$

DM: $\frac{f-a}{b-a}$, $f: \mathbb{C} \rightarrow \Omega_{0,1}$, $\exists z_0 \in \mathbb{C}$

$$a_0 = f(z_0) \neq 0, 1, \quad f'(z_0) \neq 0$$

" a_1

$$r \in [r(z_0), +\infty) \quad r \leq R(a_0, a_1).$$

COR: $f: \mathbb{C} \rightarrow \mathbb{C}$ derivata, $f \circ f$ ha un pt fisso
oppure f è una traslazione

DIM: $f \circ f, f$

$$g(z) = \frac{f(f(z)) - z}{f(z) - z}$$

$$g: \mathbb{C} \rightarrow \mathbb{C} \setminus \{0, 1\}, \quad g(z) = c \neq 0, 1$$

$$f(f(z)) - z = c(f(z) - z)$$

$$f'(f(z)) \cdot f'(z) - 1 = c(f'(z) - 1)$$

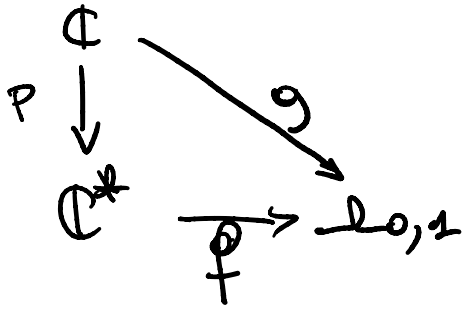
$$f'(z) (f'(f(z)) - c) = 1 - c$$

f' non ha zeri $f' \neq c$ f' è costante

$$f = az + b \quad a = 1 \quad b \neq 0$$

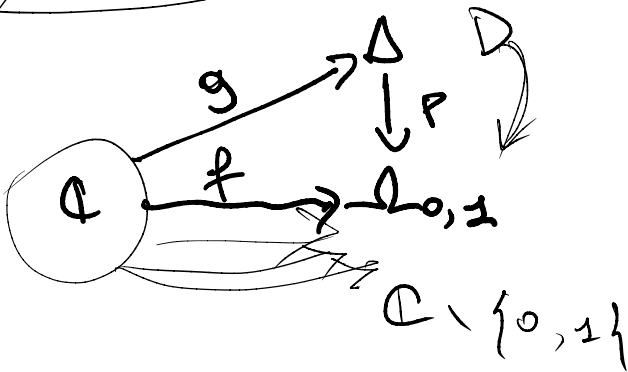
$f' \circ f$

$$\mathbb{C}^* \xrightarrow{f} \mathbb{Z}_{0,1}$$



$$g = f \circ P$$

$$\Delta \xrightarrow{P} \mathbb{Z}_{0,1}$$



$$\underbrace{P \circ g}_{=} = f$$

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